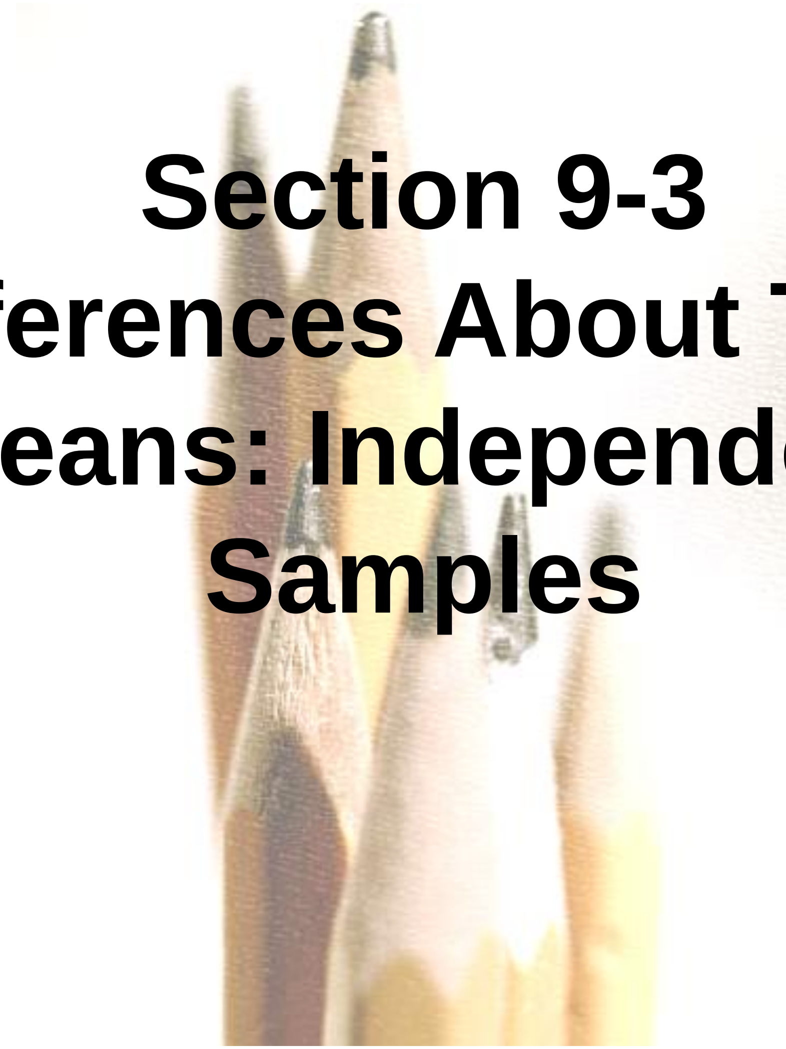
Section 9-3
Inferences About Two Means: Independent Samples



Key Concept

This section presents methods for using sample data from two independent samples to test hypotheses made about two population means or to construct confidence interval estimates of the difference between two population means.

Key Concept

In Part 1 we discuss situations in which the standard deviations of the two populations are unknown and are not assumed to be equal. In Part 2 we discuss two other situations: (1) The two population standard deviations are both known; (2) the two population standard deviations are unknown but are assumed to be equal. Because is typically unknown in real situations, most attention should be given to the methods described in Part 1.

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Part 1: Independent Samples with σ_1 and σ_2 Unknown and Not Assumed Equal

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Definitions

Two samples are independent if the sample values selected from one population are not related to or somehow paired or matched with the sample values from the other population.

Two samples are dependent if the sample values are *paired*. (That is, each pair of sample values consists of two measurements from the same subject (such as before/after data), or each pair of sample values consists of matched pairs (such as husband/wife data), where the matching is based on some inherent relationship.)

Notation

μ_1 = population mean

σ_1 = population standard deviation

n_1 = size of the first sample

$\bar{x}_1$ = sample mean

s_1 = sample standard deviation

Corresponding notations for μ_2, σ_2, s_2, $\bar{x}_2$ and n_2 apply to population 2.

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Requirements

1. σ_1 an σ_2 are unknown and no assumption is made about the equality of σ_1 and σ_2 .

2. The two samples are independent.

3. Both samples are simple random samples.

4. Either or both of these conditions are satisfied: The two sample sizes are both large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Hypothesis Test for Two Means: Independent Samples

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

(where $\mu_1 - \mu_2$ is often assumed to be 0)

Hypothesis Test - cont

Test Statistic for Two Means: Independent Samples

Degrees of freedom: In this book we use this simple and conservative estimate: df = smaller of $n_1 - 1$ and $n_2 - 1$.

***P*-values:** Refer to Table A-3. Use the procedure summarized in Figure 8-5.

Critical values: Refer to Table A-3.

Example:

A headline in *USA Today* proclaimed that "Men, women are equal talkers." That headline referred to a study of the numbers of words that samples of men and women spoke in a day. Given below are the results from the study. Use a 0.05 significance level to test the claim that men and women speak the same mean number of words in a day. Does there appear to be a difference?

Number of Words Spoken in a Day

Men	Women
$n_1 = 186$	$n_2 = 210$
$\bar{x}_1 = 15{,}668.5$	$\bar{x}_2 = 16{,}215.0$
$s_1 = 8632.5$	$s_2 = 7301.2$

Example:

Requirements are satisfied: two population standard deviations are not known and not assumed to be equal, independent samples, simple random samples, both samples are large.

Step 1: Express claim as $\mu_1 = \mu_2$.

Step 2: If original claim is false, then $\mu_1 \neq \mu_2$.

Step 3: Alternative hypothesis does not contain equality, null hypothesis does.

$H_0 : \mu_1 = \mu_2$ **(original claim)** $\quad H_a : \mu_1 \neq \mu_2$

Proceed assuming $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$.

Example:

Step 4: Significance level is 0.05

Step 5: Use a *t* distribution

Step 6: Calculate the test statistic

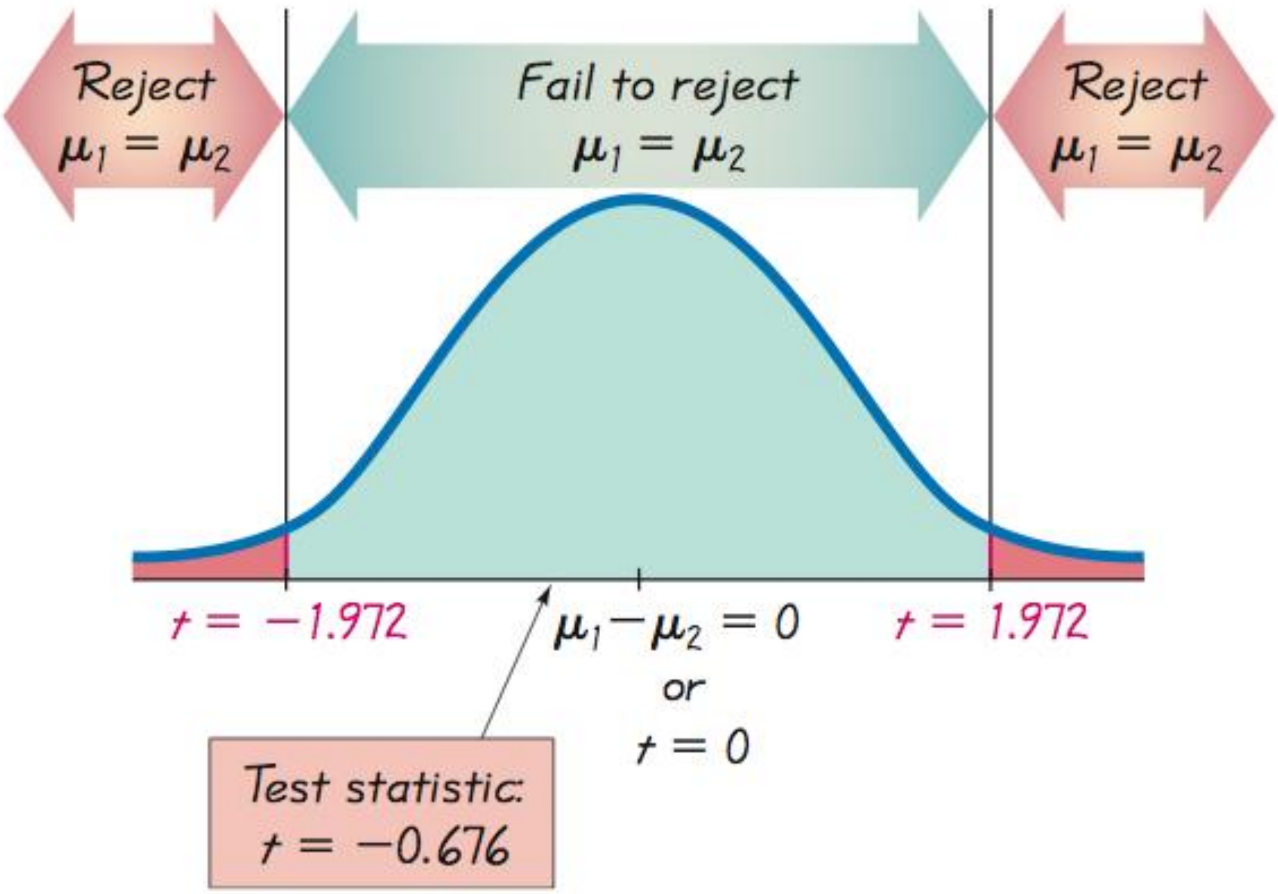
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(15{,}668.5 - 16{,}215.0) - 0}{\sqrt{\frac{8632.5^2}{186} + \frac{7301.2^2}{210}}} = -0.676$$

Example:

Use Table A-3: area in two tails is 0.05, df = 185, which is not in the table, the closest value is

$t = \pm 1.972$



Example:

Step 7: Because the test statistic does not fall within the critical region, fail to reject the null hypothesis:
$\mu_1 = \mu_2$ (or $\mu_1 - \mu_2 = 0$).

There is not sufficient evidence to warrant rejection of the claim that men and women speak the same mean number of words in a day. There does not appear to be a significant difference between the two means.

Recap

In this section we have discussed:

- **Independent samples with the standard deviations unknown and not assumed equal.**
- **Alternative method where standard deviations are known**
- **Alternative method where standard deviations are assumed equal and sample variances are pooled.**

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Section 9-4
Inferences from Matched Pairs

Key Concept

In this section we develop methods for testing hypotheses and constructing confidence intervals involving the mean of the differences of the values from two dependent populations.

With dependent samples, there is some relationship whereby each value in one sample is paired with a corresponding value in the other sample.

Key Concept

Because the hypothesis test and confidence interval use the same distribution and standard error, they are equivalent in the sense that they result in the same conclusions. Consequently, the null hypothesis that the mean difference equals 0 can be tested by determining whether the confidence interval includes 0. There are no exact procedures for dealing with dependent samples, but the *t* distribution serves as a reasonably good approximation, so the following methods are commonly used.

Notation for Dependent Samples

d = individual difference between the two values of a single matched pair

μ_d = mean value of the differences d for the **population** of paired data

$\bar{d}$ = mean value of the differences d for the paired **sample** data (equal to the mean of the $x - y$ values)

s_d = standard deviation of the differences d for the paired **sample** data

n = number of **pairs** of data.

Requirements

1. The sample data are dependent.

2. The samples are simple random samples.

3. Either or both of these conditions is satisfied: The number of pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal.

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Hypothesis Test Statistic for Matched Pairs

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

where degrees of freedom = $n - 1$

P-values and Critical Values

Use Table A-3 (*t*-distribution).

Example:

Data Set 3 in Appendix B includes measured weights of college students in September and April of their freshman year. Table 9-1 lists a small portion of those sample values. (Here we use only a small portion of the available data so that we can better illustrate the method of hypothesis testing.) Use the sample data in Table 9-1 with a 0.05 significance level to test the claim that for the population of students, the mean change in weight from September to April is equal to 0 kg.

Example:

Table 9-1 Weight (kg) Measurements of Students in Their Freshman Year

April weight	66	52	68	69	71
September weight	67	53	64	71	70
Difference d = (April weight) − (September weight)	−1	−1	4	−2	1

Requirements are satisfied: samples are dependent, values paired from each student; although a volunteer study, we'll proceed as if simple random sample and deal with this in the interpretation; STATDISK displays a histogram that is approximately normal

Example:

Weight gained = April weight – Sept. weight

μ_d denotes the mean of the "April – Sept." differences in weight; the claim is $\mu_d = 0$ kg

Step 1: claim is $\mu_d = 0$ kg

Step 2: If original claim is not true, we have $\mu_d \neq 0$ kg

Step 3: H_0: $\mu_d = 0$ kg original claim
H_1: $\mu_d \neq 0$ kg

Step 4: significance level is $\alpha = 0.05$

Step 5: use the student *t* distribution

Example:

Step 6: find values of *d* and s_d

differences are: –1, –1, 4, –2, 1

$d = 0.2$ and $s_d = 2.4$

now find the test statistic

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.2 - 0}{\frac{2.4}{\sqrt{5}}} = 0.186$$

Table A-3: df = n – 1, area in two tails is 0.05, yields a critical value $t = \pm 2.776$

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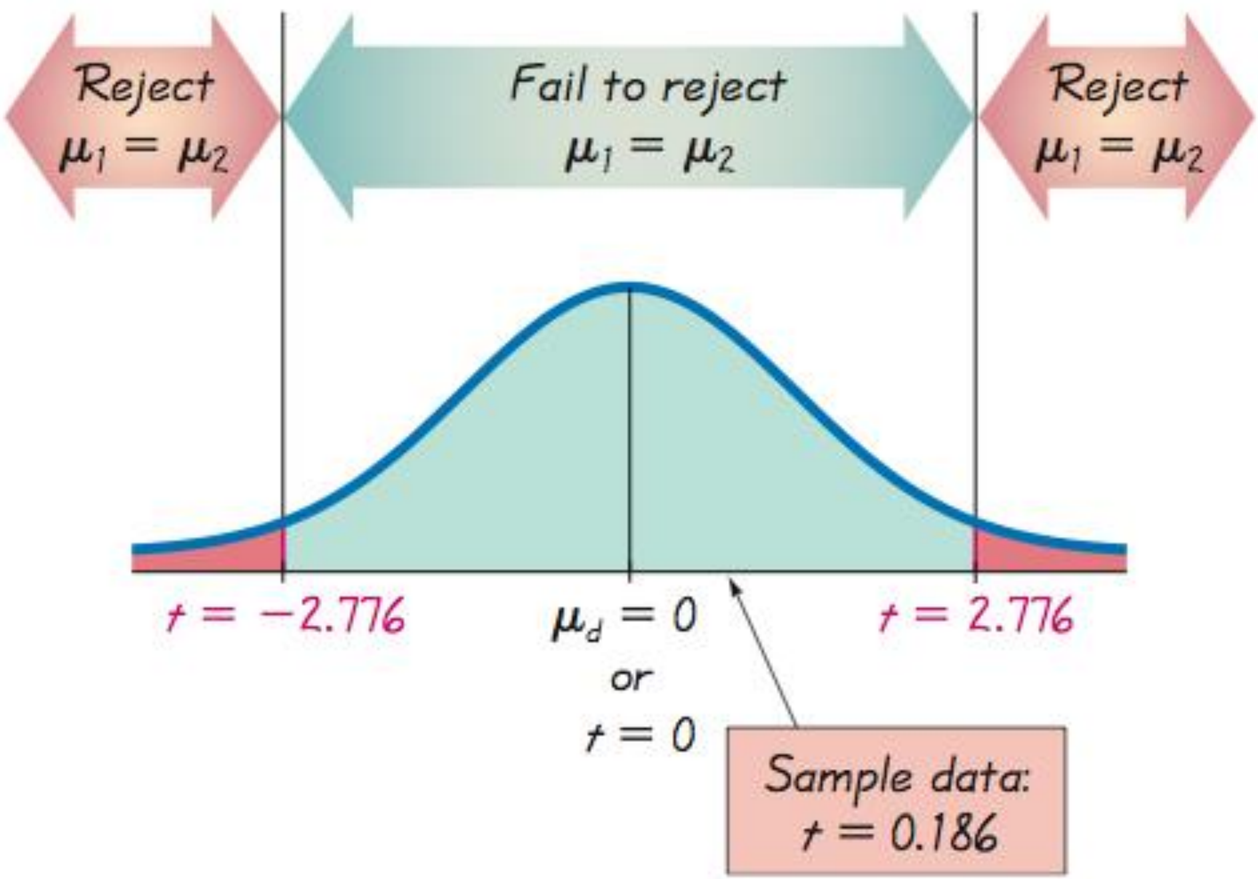
Example:

Step 7: **Because the test statistic does not fall in the critical region, we fail to reject the null hypothesis.**



Example:

We conclude that there is not sufficient evidence to warrant rejection of the claim that for the population of students, the mean change in weight from September to April is equal to 0 kg. Based on the sample results listed in Table 9-1, there does not appear to be a significant weight gain from September to April.

Example:

The conclusion should be qualified with the limitations noted in the article about the study. The requirement of a simple random sample is not satisfied, because only Rutgers students were used. Also, the study subjects are volunteers, so there is a potential for a self-selection bias. In the article describing the study, the authors cited these limitations and stated that "Researchers should conduct additional studies to better characterize dietary or activity patterns that predict weight gain among young adults who enter college or enter the workforce during this critical period in their lives."

Example:

The *P*-value method:

Using technology, we can find the *P*-value of 0.8605. (Using Table A-3 with the test statistic of t = 0.186 and 4 degrees of freedom, we can determine that the P-value is greater than 0.20.) We again fail to reject the null hypothesis, because the *P*-value is greater than the significance level of α = 0.05.

Recap

In this section we have discussed:

- Requirements for inferences from matched pairs.
- Notation.
- Hypothesis test.
- Confidence intervals.